52

JOURNAL OF CENTRE FOR PEDAGOGICAL STUDIES IN MATHEMATICS

330/Gibt

JOINT ISSUE : FOURTEENTH & FIFTEENTH
28TH AUGUST, 2005

Library
Calcutta
25/3, B. C. Road

CENTRE FOR PEDAGOGICAL STUDIES IN MATHEMATICS (CPSM)
(Regd. No. S/65536 of 1990-91, Under W.B. Societies Registration Act, XXVI, 1961)

16, South Avenue, Modern Park
Kolkata - 700 075

New Address
East Rajapur, E - Block, (Plot No - 1898, Chak Garia), Kolkata – 700 075

JOURNAL OF
CENTRE FOR PEDAGOGICAL STUDIES
IN MATHEMATICS

JOINT ISSUE : FOURTEENTH & FIFTEENTH

CONTENTS

SL. NO.	TOPIC	AUTHOR	PAGE NO.
1.	EDITORIAL		1
2.	STUDENT'S ACHIEVEMENT IN MATHEMATICS – GENDER -WISE AND STRATA-WISE ANALYSIS	CHITRALEKHA MEHERA	2
3.	HIGH ACHIEVEMENT MATHEMATICS AN THE GIRL CHILD	S. N. GIRI	6
4.	INDUCTIVE AND DEDUCTIVE LOGIC - THEIR RELATION REVISED	RITA SINHA	11
5.	SEQUENCES THE CONCEPT OF MULTIPLICATION OF WHOLE NUMBERS	SK. SAMSUL ALAM	16
6.	RELEVANCE OF HISTORY OF MATHEMATICS IN MATHEMATICS SYLLABUS	SATYABACHI SAR	23
7.	THE MATHEMATICS OF SOME PHYSICAL SITUATIONS	PIJUSH KANTI DAS	31
8.	GEOMETRICAL SIGNIFICANCE OF NEW DERIVATIVE	ASIM MUKHOPADHAY	36
9.	SOME STUDIES RELATED TO CHILDREN'S LEARNING OF MATHEMATICS	SANGHAMITRA GAYEN	39
10.	A REVIEW OF STUDIES REGARDING ACQUISITION OF GEOMETRY	ADITI GHOSH & CHANDRANI ACHARYA	41

Acc NO - 16035

EDITOR : ADITI GHOSE.
JT. EDITOR : CHANDIDAS PAL.

82

330/Gilbt

Editorial

This edition of the Journal of the Centre for Pedagogical Studies is long awaited. It comes after and during a period of transitions in education and mathematics education in particular. In West Bengal the Sarva Shiksha Abhiyan movement has gainet momentum and we now require to seriously think of a popular base for mathematics teaching. The school syllabii at all levels have undergone radical changes. The teacher education curricula of different Universities, too, have been modernised and rationalised.

All of us require to be ready to meet these changes and to devise ways and means of operationalising them. With time, we need to evaluate the new curricula and modify them. In other words, all of us, mathematics teachers and teacher educators still need to participate and evolve mathematics curricula that will match the needs of our country and help to make our nation a member of the global community. We hope that this journal will be the conduit for this transformation.

Library
Calcutta
B. C. Road

Students' Achievement in Mathematics – Gender-wise and Strata-wise Analysis

* Chitralekha Mehera

Abstract :

Achievement test in Mathematics for the students of class VIII in West Bengal, as prepared and standardized by the present investigator, was administered on 600 students comprising of urban boys, urban girls, rural boys and rural girls in equal numbers. The study revealed

(i) better Achievement of urban students than rural students,
(ii) better achievement of urban boys than rural boys,
(iii) better achievement of rural girls than rural boys,
(iv) no significant difference between the achievement of
  (a) total boys and total girls, (b) urban girls and rural girls.

Introduction :

Academic achievement of the students is the prime concern of a teacher in the classroom. Mathematics is a very useful subject for most vocations and higher specialized courses of learning.

It is generally seen that gender-difference and rural / urban location play an important role in the achievement in Mathematics among Indian students. the impact of gender-difference on the students' achievement in Mathematics and Science subjects has been well documented by many investigators in different coutries (Finlayson *et. al.,* 1979; Finn, 1980; Ganon 1980).

Investigators like Acharyulu (1978), Baskaran (1991) and others have recorded a significant gender-difference in the scores in Mathematics in favour of girls, while otherslike Katiyar (1979), Jain (1981), Rajyaguru (1991) and Rangappa (1992) failed to note any such significant gender-difference in favour of either the boys or the girls at the school level. As such, our knowledge in this respect stands incomplete since the available related records are inconclusive and often contradictory.

In view of the above, the present investigation was undertaken in order to access the achievement of the students in Mathematics at the secondary level and to note whether gender and strata had any effect on such achievement.

Material and Methods :

For the present study 600 students (14+ years) freshly promoted to class IX were used as the sample. The sample comprised, in equal numbers of urban

---

* *Institute of Science Education,Burdwan University, Burdwan*

boys, urban girls, rural boys and rural girls. In consultation with the respective class teacher concerned each of the four categories of sample was formed by taking high, average and low achieving students in an approximate ratio of 1:2:1.

Fifty students from each of the selected twelve schools (3 urban boys, 3 urban girls, 3 rural boys, 3 rural girls) of Burdwan district were used for the present purpose.

In the absence of a published standarized Achievement Test in Mathematics (Arithmetic, Algebra and geometry) for class VIII covering the prescribed syllabus of West Bengal Board of Secondary Education, the present investigator developed a standarized Achievement Test in Mathematics (ATM). Four major dimensions knowledge, understanding, skill and application were selected. Each of these dimensions was divided further into various sub-dimensions in terms of behavioural outcomes (abilities). The raliability of the test was established through test-retest and split-half methods. The r-values were respectively .96 and .72 (both significant at .01 level). The content validity and concurrent validity of the test were established. The test consisted of 150 items and a total of 160 marks.

The scores of achievement in Mathematics as obtained by the students were grouped into frequency distributions – gender-wise and strata wise.

F-test, and t-test (where necessary) were adopted in order to determine the significance of differences in the mean scores obtained by the students – gender-wise and strata-wise. For this purpose 30 students from each group of urban boys, urban girls, rural boys and rural girls were randomly selected using Tippett's (1950) table column-wise.

Results and Discussion :

F-values for gender, strata and interraction of gender and strata are given in Table – I.

Table – I

Presentation of F-values in the ATM

Source of Variation	df	SS	MS	F
Due to gender	1	138.67	138.67	.20$^+$
Due to strata	1	16969.40	16969.40	24.04*
Due to gender × strata	1	6235.22	6235.22	8.83*
Error	116	81894.93	705.99	

* significant at .01 level.
+ Not significant at .05 level.

Since, F-values were significant in respect of strata and gender × strata, t-test was done for the mean scores between urban - and rural students, urban boys and rural boys, urban girls and rural girls and rural boys and rural girls. The corresponding t-values are reflected in Table - II.

Table – II

Presentation of t-values of the concerned Mean scores of the students in ATM.

Group / Means	Urban	Rural	Urban Boys	Rural Boys	Urban Girls	Rural Girls	Urban Boys	Urban Girls	Rural Boys	Rural Girls
N	60	60	30	30	30	30	30	30	30	30
M	101.57	77.78	107.70	69.50	95.43	86.07	107.70	95.43	69.50	86.07
SD	27.13	27.11	25.73	26.72	27.15	24.84	25.73	27.15	26.72	24.84
σ_D	4.95		6.77		6.72		6.83		6.66	
t	4.81*		5.64*		1.39^{+}		1.80^{+}		2.49**	

* significant at .01 level.
** significant at .05 level.
+ not significant at .05 level.

Tables I and II reveal that urban students show better achievement in Mathematics than rural students. Significant difference in the mean scores is found in case of urban boys and rural boys and this difference is in favour of urban boys while no such significant difference is found in case of urban girls and rural girls though the average score is higher in case of urban girls. This shows that urban students are doing better than their rural counterparts regarding achievement in Mathematics.

Some of the reasons for the present findings nay be

(i) The social and cultural set up, conductive to academic achievement, of rural and urban students are far different.

(ii) Urban students are more exposed to modern life and hence get better opportunity to enhance their achievement motivation and other boostering factors.

(iii) The school - and home environments also differ for urban and rural students.

Various findings on the existance of gender difference in scholastic achievement (Satyanandam, 1969; Desai, 1979) as well as achievement in Mathematics (Harding, 1977; Murphy, 1978; Finlayson et. al., 1979) are contradictory as some investigators recorded such difference in favour of girls while others in favour of boys. In the present study no such difference is found in achievement in Mathematics between the entire sample of boys and girls as well as between the urban boys and urban girls (Table I and II). it is difficult indeed to pinpoint the cause (s) of such findings. However, rural girls show better achievement in Mathematics than rural boys (Table I and II). Within the limits of this investigation no definite reason(s) could be ascribed to explain the girls' superiority in this respect.

References :

Acharyulu, S.T.V.G., 1978. A study of the relationship among creative thinking, intelligence and school achievement. Unpublished Ph.D. Thesis, Utkal University, India.

Baskaran, K. 1991. Achievement motivation, Attitude towards problem solving and Achievement in Mathematics of standard X students in Devakottai Educational District. Ph.D., Edu., Alagappa University.

Desai, H.G. 1979. A study of classroom ethos, pupils' motivation and academic achievement. Unpublished Ph.D. Thesis, M.S. University, India.

Finlayson, D. S., Dulberg, L. and Reis. J. 1979. Sex-difference in educational attainment: A cross - national perspective. *Harvard Educ. Rev.* **49** : 477 – 503.

Finn, J. D. 1980. Sex difference in educational outcomes – a cross - national study. *Sex Roles,* **6(1)** : 9 – 26.

Ganon, C. 1980. Girls' underachievement in science *CORE,* **4(1)** : 17.

Harding, . 1977. Sex differences in examination performance at 16+. *Phys. Educ.,* 4(5) : 280 – 288.

Jain, S. S. 1981. A study of the impact of Reading on the Achievement of pupils of class VII in different school subjects. Ph.D. Thesis in Edu.,SPU.

Katiyar, P. 1979. A study of cognitive functions in relation to achievement in mathematics at high school stage. Ph. D. Thesis in Edu., Gor. U.

Murphy, R. J. L. 1978. Sex differences in examination performance - do these reflect differences in ability or sex-role stereotyping? *Educ. Rev.,* **30(3)** : 259 – 263.

Rajyaguru Mahesh, S. 1991. A comparative study of over - and under achievers in Mathematics. Ph.D. Thesis in Edu., Bhavanagar Univ.

Ranagappa, K.T. 1992. A study of self concept, reading ability in relation to achievement in mathematics of students of standard VII. Ph.D. Thesis in Edu., Bangalore Univ.

Satyanandam, B.D. 1969. A study of Socio-economic Status and Academic Achievement. Government College of Education, Kurnool, India.

Tippett, L.H.C. 1950. Technological Applications of Statistics, John Wiley, New York.

High Achievement in Mathematics and the Girl Child

* S.N. Giri

1. *Introduction*

Who are better in respect of achievement in mathematics? Boys or girls? In fact, it has been found from the results of public examinations at the secondary and past secondary stages that the average performances of boys and girls in mathematics are more or less, the same in different years. Some investigators, however, have found that boys are superior, while others have seen the superiority of girls. The mean differences between the performances of boys and girls in mathematics have been found, in many cases, not to be significant. All the answers to the question posed above are of the following types : i) boys are better than girls, ii) girls are better than boys iii) there is no significant difference between the performances of boys and girls. Each of the above conclusions has it's own interpretations.

2. *The problem*

Let us now concentrate our attention to the performances of high achievers in mathematics. The following observations may be noted : i) In the list of renowned world mathematicians, the number of woman mathematicians is quite less. ii) In mathematics based subjects, (e.g. engineering and technology), the number of girls in less. iii) In the topper lists of H.S. Examination (Science Group), the number of girl children is less in comparison to that of boys. The above observations led to the conclusion that the performances of girls are always less than those of boys at the top level.

3. *Objectives*

Under the above context, the author intended to investigate the position of girls in respect of high achievement in mathematics. It is also aimed to collect and accumulate the reflections of different kinds of mathematics-people about such achievements of the girl child.

4. *Experiences from the results of Achievement-cum-Diagnostic Test (ADTM)*

Achievement-cum-Diagnostic test is organised by Centre for Pedagogical Studies in Mathematics, a reputed organisation, every year to ascertain the potentialities and diagnose the weakness of students ranging from class IV to class XII, since last fifteen years. Every year a large number of students par-

---

**** President, CPSM, The paper was presented at EpiSTEME-I, Goa, 2004.***

ticipate in the test. The number of participant students ranges from 35 thousand to 55 thousand every year. About ten to fifteen students of each class (IV to XII), who top the list of students of each class are selected and rewarded. It has been found that about 35% of the participating students are girls. The number of boys and girls in the toppers' list in different classes and in different years been found out and given in the table below. It may be pointed out that about 30% of the students come from urban area and the rest 70% of students are from semi urban and rural area.

TABLE I :

Number of toppers in Achievement-cum-Diagnostic Test in 1996 – 2003.

Year	The number of Toppers in different classes from IV - XII																		Grand Total
	IV		V		VI		VII		VIII		IX		X		XI		XII		Total		
	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls	
1996	13	4	12	0	12	4	12	3	17	0	13	0	12	0	3	0	2	0	96	11	107
1997	19	5	11	1	9	2	8	1	12	0	12	1	8	4	2	0	2	0	83	14	97
1998	14	1	11	3	11	1	7	1	10	0	9	0	9	1	3	0	2	1	76	8	84
1999	12	0	12	1	10	2	11	0	10	1	9	0	9	0	2	0	3	0	78	4	82
2000	10	1	10	2	11	1	11	1	11	3	11	0	5	1	2	0	2	0	73	9	82
2001	7	1	7	1	7	1	6	2	8	0	9	1	7	0	3	0	3	0	57	6	63
2002	12	0	9	3	9	1	8	2	10	2	15	1	9	0	8	0	3	0	83	9	92
2003	9	4	12	0	11	0	10	0	11	2	9	1	12	0	9	0	7	0	90	7	97
Total	96	16	84	11	80	12	73	10	89	8	87	4	71	6	32	0	24	1	636	68	704

5. *Analysis*

The above table of toppers' list clearly shows that these girl children are less in number as compared to the boys. It is true that the total number of girl participants was about 35% to the total number of participating candidates. The proportion of number of boys and that of girls is 2:1 (approx.) but the proportion of the number of more-able boys and more-able girls is 9:1. The candidates of the test are both from rural, urban and semi-urban areas in proportion 2:1(approx.). The number of more able boys and girls has been found to be more in urban areas than that of the same in rural areas.

Totally speaking, it appears that, among the high-achievers, the number of girl children is less than that of boy children.

The present investigation can not claim that such generalisation is true in all cases. The results, however, can show indication that, by some reason or other, girl-children are not holding top-position in greater number, particularly, in rural and disadvantaged areas. It would be wise to investigate why such results happen in case of brighter section of students in mathematical achievement.

6. *Two case studies*

It is necessary to investigate reasons for such low number of girl toppers and ascertain the causes of relatively poor achievement. First of all, it is necessary to know, what the students, teachers and administrators are thinking about this.

The following steps were undertaken.

i) An instant talk competition was arranged by the author, 12 students of class XI, and each one from twelve schools of which six were from Boys' and other six from girls' schools of Kolkata. Each student was asked to talk on the topic, " Why are the girls weak in mathematics?" The boys and girls gave relevant arguments in support of their answers. The responses revealed a lot of ideas in regard to so called weakness of girls in mathematics.

ii) A second step was taken. Twenty experts of whom ten are males and ten are females were selected. The experts were teacher-educators, teachers of mathematics from schools and colleges, and educational administrators. All of them were participants of a seminar on mathematics teaching organised by Centre for Pedagogical Studies in Mathematics. Each of the twenty participants were asked to state reasons on the issue 'Why is the number of girls less than that of boys among the high achievers in mathematics?' The group of experts took the issue enthusiastically and gave their opinions and comments regarding such achievement of girls.

7. *Why is the number less?*

The opinions expressed in the talk competition by the students, both boys and the girls were collected. Again the opinions and comments expressed by the experts were also taken into account. Finally a list was prepared, which according to the above respondents were reasons for low achievement of girls. These reasons were many, diverse and varied. It is to be noted that these reasons are not exhaustive. They, however, give a glimpse of thoughts cherished by the learners as well as the experts about such low achievement of girls.

8. *Reasons for low achievement of the girl child.*

An analysis of the opinions of learners of experts revealed the following:

i) Most of the male experts opined that majority of the girl children are 'naturally' weak in mathematics and in mathematical reasoning, while most of the female experts did not agree to such a comment.

ii) The guardians do not usually encourage the study of mathematics for their daughters.

iii) The guardians normally think at the very beginning that the girl children are poor in mathematics.

iv) The girl children have lesser interest, attitude and attention in mathematical thinking.

v) In the matrimonial market the would-be-husbands and their parents prefer that the prospective bride should be conversant in fine arts (dance, music, drawing etc.) and the humanities, rather that in mathematics, science and engineering.

vi) In rural areas, the negligence of the guardians and the society to the girl child in their study of mathematics and science is highly discouraging.

vii) The girls have to do household work in addition to their study.

viii) The girls are prohibited to go outside and discuss the lessons with their boy friends, and male tutors and teachers.

ix) The experts agreed that there are many bright girl children also, who are learning mathematics with joy and enthusiasm in a better way than many of their boy classmates.

x) The expert group also opined that, given proper opportunity and encouragement, the girl child can excel in the same way as the boys.

xi) According to them it is encouraging that the girl children are doing better now as compared to earlier days.

xii) The number of girl students in a particular sample is less and hence the number of high achiever girls is less.

xiii) Boys are more efficient in the examination hall than the girls and hence boys get greater score in mathematics than girls.

9. *Observations and their interpretations.*

First of all, it is to be admitted that the observation obtained from the results of ADTM for boys and girls are confined only to the population taken in the study. Thus, the observation that the number of girls is less than that of the boys in respect of high achievement in mathematics can not taken as a generalised conclusion. In fact, we often find many bright girl students who are shining in future as eminent mathematicians. But many facts, figures and findings as stated in our paper earlier, show that the number of girl-toppers is less than that of the boy-toppers in regard to achievement in mathematics.

Some opinions for this underachievement of girl child have been given above. It is to be carefully examined to what extent they can be accepted as valid reasons for under achievement of the girl child. It has been seen that 'bright' girl students in mathematics are less in rural areas than in urban areas.

It can thus be said that as socio-cultural conditions improve, the consciousness of gender equality also improves, and as such, the girl child gets greater motivation to do better.

Another significant finding is that such bright girl students are less in upper grades than in lower grades.

At lower secondary stages the boys and girls get, to some extent, equal educational opportunity. But, at upper secondary and higher secondary stages, the attention of guardians are focussed more on boys than girls. Again the parents also take greater attention to see that their daughters take up humanity subjects rather than science subjects. Thus the issue of such under achievement ultimately reduces to social neglect to the girl child. In fact, in many cases, parents give more attention to boys, because they think that boys will get jobs and will help family in future, so they select the boy child for study of science and mathematics, and provide maximum possible facilities to them. The whole atmosphere of learning mathematics for the girl child is in most cases not congenial for mathematical study. Again, the would-be-husbands and his family members prefer the bride to be conversant in fine arts, music, dance, cooking, humanities etc. rather than being engaged in the study of mathematics.

Let us consider the simulation in the classroom also in co-education classes, the teacher has in many cases the pre-occupied idea that boys are superior to girls, and so asks more questions to boys and lesser number of questions to the girls. The girls, on the other hand, being naturally shy and timid, do not intend to respond properly. The spontaneous intention of learning is thus thwarted. Being deprived of getting encouragement from parents, teachers, boy friends, family members and the society, the girl child herself starts thinking that she is inferior to the boy child in the study of higher mathematics.

It must, however be admitted that the above gloomy situation of learning mathematics is not always true. But, in most cases, the under-achievement of the girl child in mathematics, in case of occupying top-positions, is due to the negative attitude of gender-difference to the girl child by the school, family and the society.

Inductive and Deuctive Logic – Their Relation Revisited

* Rita Sinha

Introduction :

Mathematics is the science of logical reasoining. Reason is the vehicle of knowledge. Knowledge is the central theme of human existence. The capacity for knowledge distinguishes man from all other living creatures. Plants have bare feelings, animals have rudimentary consciousness, devoid of reason, but human beings have the capacity for rational thinking and systematic knowledge. Knowledge is the powerful tool for solving human problems.

Knowledge is by its very nature well ordered, structured and systematic. Systematic knowledge is logical and scientific. When knowledge is concerned with specialized investigation of a particular aspect of nature, it is known as science. There are three main distinguuishable aspects of nature, that is matter, life and mind. Science may be broadly classified into three main classes, namely Physical Science, Biological Science and Behavioural Sciences. The principles, laws, paradigms and theories of science are developed through inductive or deductive logic.

Mathematical equations :

Scientific knowledge tallies with the actual phenomena of nature. Scientific study reveals that a particular phenomenon of nature is basically structureed and that there is systematic relationship between these structured aspects of a phenomenon. This can be precisely stated in exact quantities and these relationships can be precisely stated in mathematical equations. Ordinary language is incapable of stating the relationships between quantified aspects of phenomena in exact expressions. A mathematical equation, on the other hand, describing the relationship between the quantized aspects of a phenomenon is not only exact but also brief.

The method of acquiring knoledge can be classified under the categories like (a) authority, (b) tradition, (c) experience, (d) deductive reasoning, (e) inductive reasoning and (f) scientific method. Charles Darwin integrated the most important aspects of the inductive and deductive methods which he called inductive - deductive method or scientific method. The theory developed with the help of scientific method can be stated in Mathematical equation.

---

* *Rita Sinha, Department of Education, Calcutta University*

When we study Physics, we find that all aspects of physical phenomenon can be most presisely and satisfactorily stated in mathematical terms. There are exact equations for heat, light, electricity and magnetism. For instance, Einstein's famous equation $E = mc^2$ very beautifully states the exact nature of energy. Boyle's law which states that at constant temperature, the product of the volume of a certain mass of a gas and its pressure is constant, can be written as PV = Constant. The first law of Thermodynamics may be expressed as $E_2 - E_1 = q - w$ or $\Delta E = q - w$; which means that if a system absorbs heat 'q' to which its internal energy increases from 'E_1' to 'E_2' and the system does the work equal to 'w' on its surroundings. Here ΔE is increase in internal energy of the system. Similarly preparation of hydrogen can be explained with the help of the formula $Zn + H_2SO_4 = ZnSO_4 + H_2$.

We find through the study of Biological Science that the living phenomena are also highly complex and structured. The relationship between the various aspects of a living organism or a living cell can be stated with a great deal of precision with the help of chemical formulae and mathematical equations. The formula $SER = D \times V \times K \times SHR$ proposed by Hull, explains the learning process to a great extent. Here SER is the reaction potential, D is the drive, V stands for dynamism of the signaling stimulus trace and K is the incentive reinforcement. Thus mathematics is a language. It is the science of reasoning.

The variables of relationships can be represented in some graphic forms. A paradigm for Pavlovian conditioning can be shown in the following diagram.

$$\begin{tikzcd}
S_1 \arrow[r] & R & S_2 \arrow[dr, dotted] & & S_2 \arrow[r] & R \\
& & S_1 \arrow[r] & R & &
\end{tikzcd}$$

Mathematics is the science of measurement, quantity and magnitude. It deals with quantitative facts and relationships as well as with problems involving space and time. It may be regarded as a highly disciplined mode of thinking. Where ever there is structure, relationship, regularity, systematic variation, there is mathematics.

The various phenomena of Nature are structured and patterned with logical relationship between their elements. The relationships between the elements of some of the structured systems and subsystems of Nature are so well-ordered that they may be precisely stated in mathematical language. Hegel had reflective understanding of this aspect of Nature, and therefore, he maintained that "logic" or reason is immanent in the entire universe. the development of Nature is nothing but the movement of logical categories. The whole world seems to be mathematically designed. Sir James Jeans (1942) believed that "God is a supreme mathematician who created this well ordered and systematic universe". Pope, therefore, rightly said, "All are but parts of one systematic whole; of which world is body and God its soul".

Mathematics is the science of reasoning, which enables an individual to know the laws of nature. When we study social science, we find that human personality is not like an isolated island in a vast ocean of emptiness, but that he is a member of a well integrated society. Sociology as a science is able to state the various social process in exact equational structures. In fact, an individual is born with an in-built mechanism in the nervous system that enables him to think in mathematical terms, behave mathematically and understand mathematical concepts.

Mathematics is the main piller on which all the sciences are based. In this sense mathematics may be regarded as 'Science of all Sciences'.

Duductive and Inductive Logic :

Aristotle developed the syllogism, which can be described as a thinking process in which one proceeds from general to specific statements by deductive reasoning. It tests the validity of any given conclusion or idea by proceeding from known to unknown. The syllogistic reason consists of (I) a major premise based on a self-evident truth of previously established fact or relationship, (II) a major premise concerning a particular case to which the truth, fact, or relationship invariably applies; and (III) a conclusion. If the major premise or minor premise are true, then the conclusion arrived at is also true. A simple example of deduction can be given through the following proposition :

I) All men are mortal;
II) Mr. X is a man;
III) Threrefore Mr. X is mortal.

Deductive reasoning can identify new relationships as one proceeds from known to unknown, but it cannot be regarded as a self-sufficient method for securing reliable knowledge.

Inductive reasoning, or induction, moves from particular to the general. In order to be absolutely certain of an inductive conclusion, all instance must be observed. Therefore in inductive reasoning, a conclusion is reached by observing instances and generalizing from instances to the whole phenomenon. For example, the propositions are :

I) Mr. A is mortal
II) Mr. B is mortal
III) Therefore, all men are mortal.

In order to be absolutely certain of an inductive conclusion, all instances must be observed. Under Baconian system of reasoning it is known as perfect induction.

However, both deductive and inductive method have advantages and limitations. If the premises are unreal or erroneous, the conclusion arrived at will not be valid in deduction. the conclusion reached by imperfect inductive reasioning will contain information which is not present.

Karl popper (1957) remarks that the belief that inductive inference which is based on many observations is a myth. In his book 'The Logic of Scientific Discovery' he encapsulates the essentials of his method of falsification. he criticises the view that inductive inference provided a universal method for obtaining the general laws of Nature through maximum number of empirical observation. When a hypothesis is tested and accepted and not falsified, the evidence for the acceptence is not deductively conclusive like a mathematical equation, for example $x^2 - y^2 = (x + y)(x - y)$. Infact, the evidence merely describes strong inductive confirmation.

Scientific method involves a double movement of reasoning from induction to deduction. In its simplest form, it consists of working inductively from observations to hypothesis and then deduvtively from the hypothesis to the logical implications of the hypothesis in relation to what is already known. In modern terminology this method is described as Inductive - Hypotico-Deductive' method.

The paradigm or theory obtained by inductive - hypothetico deductive method are provisional, transitory, temporary and can be valid only today. Thomas Khun gave an organic character to science through his book "The structure of Scienctific Revolutions" (1982). He aptly observed that paradigm has a certain life span, and its development conforms to certain predictable consequences. According to Kerlinger (1978), a theory is a set of interrelated constructs (concepts), definitions and propositions that present a systematic view of phenomena by specifying relations among varriables, with the purpose of explaining and predicting the phenomena.

Earl Babbie (2004) observed that ideographic and Nomothetic reasoning can be found in everyday life. In the ideographic mode it is written. "He is like that because his father and mother kept giving him mixed signals". In nomothetic mode it is written as "Teenage boys are like that". In ideographic mode, we have a seemingly complete explanation for the behaviour of the one boy in question. In the nomothetic mode, we have a simpler, more general explanation,which wouldn't necessarily be true of all teenage boys but portrays a general pattern.

A Libin and J. Cohen - Mansfield (2000) suggested that both ideographic and nomothetic approaches may be combined in soial science research.

According to A.K. Sinha (1973), empirical knowledge is not derived exclusively by the metheods of logic and scientific method. Empirical knowledge is obtained through interpretations, imaginations, specifications and conjectures. The creative imagination and breadth of insights of creative thinkers play very important roles in the formulation of empirical knowledge.

The author of this paper is of the opinion that the telic individual is imaginative and creative. He is able to appreciate the order, symmetry, structure and pattern in this universe. Wherever there is structure, relationship, regularity and systematic variation, there is logic and mathematics. To recognize this phenomenon we need some knowledge of symbols and formulae and mathematical skills; but above all we need creative imagination.

Conclusion :

To conclude then, all events in nature are, at least to a degree, ordered, structured, regular, lawful and predictable. This order, structure, predictability and regularity of nature can be discovered through logic and mathematics. A logician relies upon empirical observation. Both inductive and deductive method as well as creative imagination is needed for finding the truth. Many phenomena of the universe can be predicted through mathematical calculations. On 24th October, 1995, people watched the solar Eclipse and the 'Diamond Ring' during its totality. But the exact date and time of the eclipse was calculated much earlier by scientists, using mathematical calculations. Logicians find explanations for observed events and relationships for specific phenomena with maximum objectivity. On the basis of such relationships they make deductions and predictions.

References :

1. Babbie, Earl (2004), the Practice of Social Research, Thomson Asia Pte. Ltd. Singapore, P. 24.
2. Jeans, J. (1942), Physics and Philosophy,P. 175.
3. Kerliger, F.N. (1978), Foundations of Behavioural Research, New York, Hold, Rinehard and Winston, P. 9.
4. Kuhn, Thomas (1962), The Structure of Scientific Revolutions, University of Chicago Press.
5. Popper, K.R. (1961), the Logic of Scientific Discovery, New York, Science Editions, Inc. P. 111.
6. Sinha, A.K. (1973), Human Civilization as the Unfoldment of Logical Principles and Scientific method, Kurukshetra University Research Journal Vol. VII, No. 1 & 2, P. 152.
7. Sinha, R. (1996), Mathematics – The Science of All Sciences, Deepan, P - 9.
8. Sinha, R (1999), Education at the Dawn of New Millennium, New Central Book Agency, Kolkata, pp. 25-32.

Seqeuncing The Coccepts Of Multiplication Of Whole Numbers

* Sk. Samsul Alam

Introduction :

Mathematics is a compulsory subjct of any modern school curricula because it is an inseparable part of our daily life. Kothari (1966) said, "Proper foundations in the knowledge of the subject (mathematics) should be laid at the school." So mathematics education should be stressed at an early stage. M.B. Buch (Ed) (1991) said, "Analysis of various board results by the N.C.E.R.T. has shown that a large number of students fail in mathematics. Researchers need to study this problem state wise and identity reasons. Some studies have been done but they are sporadic, not systematic." According to Piaget (1973), "Every normal child is capable of learning mathematics." So there is some thing wrong in our teaching learning paradigm of mathematics. Many factors are responsible for the above unwanted situation. Lacuna in content organization is one of the important factors. R. Chilana (1984) said, "Mathematical concepts should be graded in such a way that there is a proper sequence in their growth and development." In two separate papers Samsul (2003) has studied the sequencing of the concepts of addition and subtraction of whole numbers. The major concepts of multiplication of whole numbers and their sub concepts will be presented in this paper.

Objective of the Study :

The Objective of the study is to identity different sub-concepts and their sequential order of multiplication of whole numbers.

Total Frame Study :

The major concept multiplication of whole numbers has been divided into 25 sub-concepts (1,2,........ 25) which are given below in TABLE-1. Each sub-concept has been divided into minor sub-concepts. Total 409 minor concepts have been identified. The number of minor concepts under each of the 25 major sub-concepts is shown in TABLE -2. The major concepts under major sub-concept no. 2 have been given in this paper.

*** Centre for Mathematics Education (Deparment of Mathematics),**
Visva-Bharati, Santniketan - 731235, e-mail: syamal_123@yahoo.co.in

TABLE - 1
Multiplication of Whole Numbers (Major Concept)

Sub-concept sl. no.	Sub-concepts
1.	Concept of multiplication and its symbol.
2.	Multiplication where both multiplicand and multiplier are one-digit number.
3.	Multiplication by zero.
4.	Multiplication where multiplicand is two-digit number and multiplier is one digit number where the product is two-digit number.
5.	Multiplication where multiplicand is two-digit number and multiplier is one-digit number where the product is three-digit number.
6.	Multiplication where multiplicand is three-digit number and multiplier is one-digit number where the product is three-digit number.
7.	Multiplication where multiplicand is three-digit number and multiplier is one-digit number where the product is four-digit number.
8.	Multiplication where multiplicand is four-digit number and multplier is one-digit number where the product is four-digit number.
9.	Multiplication where multiplicand is four-digit number and multiplier is one-digit number where the product is five-digit number.
10.	Multiplication where multiplicand is five-digit number and multiplier is one-digit number.

12.	Multiplication where multiplicand is more than six-digit number and multiplier is one-digit number.
13.	Multiplication where multiplicand is one or two-digit number and multiplier is two-digit number within 20 (using multiplication table from 11 to 20).
14.	Multiplication where multiplicand is upto two-digit number and multiplier is two-digit number (two step multiplication).
15.	Multiplication where multiplicand is three-digit number and multiplier is two-digit number (two step multiplication).
16.	Multiplication where multiplicand is four-digit number and multiplier is two-digit number (two step multiplication).
17.	Multiplication where multiplicand is five-digit number and multiplier is two-digit number (two step multiplication).
18.	Multiplication where multiplicand is six-digit number and multiplier is two-digit number (two step multiplication).
19.	Multiplication where multiplicand is any greater than six-digit number and multiplier is two-digit number (two step multiplication).
20.	Multiplication where multiplicand is upto three-digit number and multiplier is three-digit number (three step multiplication).
21.	Multiplication where multiplicand is four-digit number and multiplier is three-digit number (three step multiplication).
22.	Multiplication where multiplicand is five-digit number and multiplier is three-digit number (three step multiplication).
23.	Multiplication where multiplicand is six-digit number and multiplier is three-digit number (three step multiplication).
24.	Multiplication where multiplicand is any greater than six-digit number and multiplier is three-digit number (three step multiplication).
25.	Multiplication where multiplicand is any number and multiplier is more than three-digit number (more than three step multiplication).

TABLE - 2

Detailed List of Different Levels of Minor Sub-Concepts

Major sub-conc-ept no.	No. of first level minor sub-concept	No. of second level minor sub-concept	No. of third level minor sub-concept	Total minor sub-concept
1.	3	-	-	3
2	5	6+6+1+1+1	-	15
3	4	5 each	15 each for every first level sub-concept	60
4	9	6+6+1+1+6+6+1+1+1	-	29
5	9	6+6+1+1+6+6+1+1+1	-	29
6	9	6+6+1+1+6+6+1+1+1	-	29
7	9	6+6+1+1+6+6+1+1+1	-	29
8	9	6+6+1+1+6+6+1+1+1	-	29
9	5	6+6+1+1+1	-	15
10	5	6+6+1+1+1	-	15
11	5	6+6+1+1+1	-	15
12	5	6+6+1+1+1	-	15
13	6	1+6+6+1+1+1	-	16
14	5	6+1+6+1+1	-	15
15	5	6+1+6+1+1	-	15
16	3	6+1+1	-	8
17	3	6+1+1	-	8
18	3	6+1+1	-	8
19	3	6+1+1	-	8

20	3	6+1+1	-	8
21	3	6+1+1	-	8
22	3	6+1+1	-	8
23	3	6+1+1	-	8
24	3	6+1+1	-	8
25	3	6+1+1	-	8
			Grand Total =	409

The researcher has detailed out the major sub-concept number-2 only (out of the 25 major sub-concepts) for economy of space. The questions for evaluation of the minor sub-concepts of the major sub-concept number -2 have not been presented both in language and picture form for the same reason. It may be supplied gladly to the interested readers on request. Here only the naming of minor sub-concepts is shown to minimize the space.

Detailing of major sub-concept no. 2

2. Multiplication where both multiplicand and multiplier are not one-digit number.

The first level minor sub-concept of sub-concept no. 2

2.1 Vertical multiplication.
2.2 Horizontal multiplication.
2.3 Multiplication after transformation from horizontal form to vertical form.
2.4 Multiplication after transformation from vertical form to horizontal form.
2.5 Commutative property of multiplication.

The second level minor sub-concept of 2.1

2.1.1 Expression of the problem in mathematical language (✱✱✱✱).
2.1.2 Expression of the problem in common language from mathematical language (✱✱✱✱).
2.1.3 Multiplication (✱✱✱✱).
2.1.4 Solution of the given problem (✱✱✱✱).
2.1.5 Solution of the given problem and writing proper answer (✱✱✱✱).
2.1.6 Formation of problem and its solution (✱✱✱✱).

The second level minor sub-concept of 2.2

2.2.1 Expression of the problem in mathematical language(***).

2.2.2 Expression of the problem in common language from mathematical language(*).

2.2.3 Multiplication (*).

2.2.4 Solution of the given problem (*).

2.2.5 Solution of the given problem and writing proper answer (***).

2.2.6 Formation of the problem and its solution (****).

The second level minor sub-concept of 2.3

2.3.1 Multiplication after transformation from horizontal form to vertical form (****).

The second level minor sub-concept of 2.4

2.4.1 Multiplication after transformation from vertical form to horizontal form (****).

The second level minor sub-concept of 2.5

2.5.1 Communitative properly of multiplication (*).

* – represents the concept which is introduced in the prescribed books of West Bengal Board of Primary Education and N.C.E.R.T.

** – represents the concept which is introduced in the prescribed book of N.C.E.R.T. only.

*** – represents the concept which is introduced in the prescribed book of West Bengal Board of Primary Education only.

**** – represents the concept which is not introduced in the prescribed books of West Bengal Board of Primary Education and N.C.E.R.T.

Conclusion :

After analyzing the major concepts multiplication of whole numbers which consists of 409 minor sub-concepts. It is interesting to note that out of these 409 minor sub-concepts only 88 minor sub-concepts have been introduced in the prescribed books of West Bengal Board of Primary Education. Similarly N.C.E.R.T. text books have also considered 66 minor sub-concepts out of 409 minor sub-concepts. 321 minor sub-concepts have been totally left out in both West Bengal Board of Primary Education and N.C.E.R.T. In this paper, 15 minors sub-concepts of the major sub-concept : multiplication where both multiplicand and multiplier are one-digit number have been particularly presented. It has been found that out of these 15 minor sub-concepts only 6 have been considered in the prescribed book of West Bengal Board of Primary Education. Similarly in N.C.E.R.T.

text book, 4 minor sub-concepts out of 15 have been considered. So it is a major gap in our text books of mathematics. It may be specially mentioned that the major sub-concepts (8,9,10,11,12,17,18,19,23,24) have totally been neglected. Multiplication after transformation from horizontal form to vertical form has not been duly weighted in the prescribed books of West Bengal Board of Primary Education. Multiplication after transformation from vertical form to horizontal form has totally been neglected.

It is hoped that this study will help to decrease the gap in teaching multiplication and make the syllabus pertaining to multiplication of whole numbers systematic and scientific. It will also help to diagnose the particular areas of weakness of the student and in planning for necessary remedial measures.

Further Study :

The diagnostic study may be taken on large number of samples of class II, III, IV on the multiplication of whole numbers.

I am thankful to Dr. A. Sensarma, Siksha-Satara, Visva-Bharati and Prof. S.K. Samanta, Department of Mathematics, Visva-Bharati for their help and guidance in preparing this paper.

References :

I. Buch M.B. (Ed) (1991), Fourth Survey of Research in Education, 1983-88, Vol-1, P-24, NCERT, INDIA.

II. Chilana R (1984), Teaching Elementary School Mathematics, The Primary Teacher, Vol- IX, No.-4, P - 31, NCERT, INDIA.

III. Kothari D.S (1966), Report of Education Commission, 1964-66, Ministry of Education, Government of India, P-197.

IV. Piaget J. (1973), To Understand is to Invent, Viking, NEW YORK.

V. Samsul (2003), Sequencing the concepts of subtraction of whole numbers, Journal of Centre for Pedagogical Studies in Mathematics, Thirteenth issue, PP 5-13.

VI. Samsul (2003), Sequencing the concepts of Addition of whole numbers, Indian Journal of Mathematics Teaching, Vol-XXIX, Nos. 1 & 2, PP 13-23.

Relevance of History of Mathematics in Mathematics Syllabus

* Satyabachi Sar

1. Introduction

Once Napolean said that the progress and development of mathematics is closely related to the overall development of a nation. A person concerned with mathematics education knows that the progress and development of mathematics depends upon the appropriate amount of mathematics learned at every stage of study in an effective manner. Certainly, this is again dependent on curricula and syllabii of mathematics at different stages of study. Here we would like to consider mathematics syllabuses at the higher secondary stage and under graduate level and enumerate the reasons for inclusion of history of mathematics in the mathematics syllabus.

2. Historical Background

Truely speaking, movement for inclusion of history of mathematics in mathematics syllabus is not a new one. Long ago, in 1890, renowed mathematical historians like David Eugene Smith and Florian Cajori tried to draw the attention of the mathematical community to the history of mathematics. But wide spread international movement for it began only in 1970 and a distinct awareness was observed only at the second International Congress on Mathematical Education (ICME) held in Exter, UK in 1972. At ICME-2, a Working Group on 'History and Pedagogy of Mathematics' was organised. The work of this group was continued at ICME-3, held in Karlsrule, Germany in 1976 with three sessions under the title of 'History of Mathematics as a critical tool for curriculum design' in which several participants gave useful talks. At the meeting 'the importance and wide spread interest in historical - pedagogical studies in mathematics' was felt necessary and a resolution was forwarded to International Commission on Mathematical Instruction (ICMI) proposing setting up of a system to ensure regular sessions at future ICME's on the relation between history and pedagogy of mathematics. ICMI Executive Committee welcomed the proposal and subsequently approved the new Study Group entitled 'International Study Group' on Relation between History and Pedagogy of Mathematics, cooperating with the International Commission on Mathematics Instruction', which generally came to be known as 'HPM', in short.

The 'principal aims' of HPM among other things are the following :

Satyabachi Sar, M.B.B. College, Agartala, Tripura

(i) To promote international contacts and exchange information concerning :

  (a) Courses in history of mathematics in universities, colleges and schools.

  (b) The use and relevance of history of mathematics in mathematics teaching.

  (c) Views on the relation between history of mathematics and mathematics education at all levels.

(ii) To facilitate access of materials in history of mathematics and related ares.

(iii) To promote awareness of relevance of history of mathematics for mathematics teaching among mathematicians and teachers.

Also International Congress of Mathematicians (ICM) held in 1978 had a session, namely, (i) How can you use history of mathematics in teaching mathematics in primary and secondary schools and (ii) The relevance of philosophy and history of science and mathematics for mathematical education.

Next ICME's also took necessary steps for promoting history in mathematical education.

In India there were sporadic efforts for the movement. Department of pure Mathematics of Calcutta University in 1972 introduced 'Mathematical Logic and History of Mathematics' as one of the post graduate branches of study in the department. Many seminars on history of mathematics have been organised by different institutions. But all these efforts had very little effect as a whole.

However, the movement for inclusion of history of mathematics in the mathematics syllabus has been gaining momentum day by day all over the world. Different countries are taking positive steps towards it. It may be mentioned here that Sweden and Denmark have already introduced history of mathematics in the mathematics syllabus.

3. Different aspects of framing mathematics syllabus :

In making teaching of mathematics at a particular stage more effective, more meaningful and more interesting we need appropriate quantity and quality of the syllabus of mathematics at that stage.

During framing the syllabus of mathematics we should keep in mind the aims of mathematics learning. Mathematics should not be taught only as a set of rules and formulae that students have to memorize and a set of problems that they have to solve; but mathematics is more. Usually objectives of mathematics learning are knowing certain mathematical facts and being able 'to think correctly, logically and scientifically'. I think, we should include more in this list. Among other objectives, the main objective may be to doubt, to inquire, to see alternatives and also to construct new perspectives and conviction. And objectives of teaching mathematics should be to discover new facts about one's self, society and culture.

Here I want to point out that there is often a misconception in teaching mathematics, viz. the belief that mathematics can be taught effectively and meaningfully without relation to culture. But it is to be borne in mind that culture influences the way people see things and understand concepts.

It is to be noted that the view about mathematics is changing continually, the facts of mathematics are changing, its interdisciplinary linkage in phenomenal and sociocultural dimensions are important. Mathematics is basically a human activity – arising from experiences but working certainly in minds.

While considering the syllabus of a subject, in general, and of mathematics, in particular, at a particular level one has to keep in mind the syllabus under consideration : i.e. of what level and from which level it is transiting and to what level it will diverge. This is because, continuity and logical coherence are two important criteria of consideration of syllabus.

With the perspective and background stated above I think history of mathematics should be included in both higher secondary stage and undergraduate stage in appropriate extent in the syllabuses of mathematics.

Specific reasons for this proposal are given below explicity.

4. *Reasons for inslusion of history of mathematics in mathematics syllabus :*

One way give many reasons in favour of the proposal, but here we mention only important ones.

4.1 Study of history of mathematics will impart a sense of completeness in the syllabus. It will help learners to get deeper insight, new power and complete understanding of proper advancement in mathematical ideas. Many mathematical ideas and concepts developed under pluralistic enterprises of great luminaries have much bearing on the study of mathematics and study of history of mathematics will certainly enable learners to understand that significance.

Progress and development of science in general and mathematics in particular have their own history. This history in esential to understand to perspective and importance , specially in society. Science without history sometimes leads us to underestimate its impact and keeps us ignorant about the sustained struggle of people behind it. Naturally, this cannot be the method of complete learning. Historical presentation is necessary for the pupose of relating the development of science to the development of civilization and of revealing science as a great stream rather than a static mass. Science without history is science without its fame and flavour.

The same applies to mathematics, too. G.F. Simonns rightly said, 'Mathematics without history is stripped of its greatness; for like the other arts, mathematics is one of the supreme arts of civilization – it derives its grandeur from the fact of its being human creation.'

Every nation is interested in the history of culture and civilization. Mathematics is often rightly considered as a mirror of civilization. Reaserches in mathematics are reliable records of intellectual struggle and progress. So, through history of mathematics we are able to look into the past and trace the line of intellectual development which links past with the present and as such, guides us to think for the future.

Teaching of mathematics without a cultural content, by claiming that it is absolute, abstract and universal, is believed by many educators to be the main reason for the alienation and failure of the majority of students in mathematics. They strongly believe that teaching mathematics through cultural relevance and personal experiences helps learners know more about reality, culture, society and themselves. That will help them become more self - confident. It will help them build new perspectives and synthesis, and seek new alternative.

Florian Cajori said, 'History of mathematics is important also as a valuable contribution to the history of civilization; so, history of mathematics is more relevant to India which has continuity of civilization with her contribution of about 5000 years of philosophy. Thus of arts, literature, and science. Thus the study of history of mathematics is more meaningful and more urgent in our Indian context.

Leibnitz once claimed that mathematics should be presented as a history of ideas and problems and rightly so. This would make the study of mathematics more interesting, more meaningful and more enjoyable. If one goes into the depth of the subject, one would certainly agree with J.W.L. Glaisher that 'no subject loses more than mathematics to dissociate it from its history.'

The value of historical knowledge to the teachers is immense. Students' interest in their study may be greatly increased if the solution of a problem or the proof of a theorem be interspersed with historical remarks and anecdotes. F.Cajori rightly said, 'In his historical talk it is possible for the teacher to make it plain to the students that mathematics is not a dead science but a living one in which steady progress is made.'

Besides, study of history of mathematics promotes many benefits to teachers and students. We state them in brief :

4.2 (a) Benefits to the teachers :

A knowledge of arguments and discussions among great mathematicians might induce healthy skepticism and discussion in the class room and may lead to further grasp of principles. A knowledge of history of mathematics will enable a teacher to convey to the students that much of what is taught to-day as finished products are a result of centuries of searching or of spirited controversy.

A deep knowledge in history of mathematics is a valuable asset to a teacher because a possession of this will help him/her to make innovations in teaching and make it interesting to the students. The Historical background can help him/her to understand inherent difficulties of some concepts and by understanding these he/she can improve his/her power of communication which in turn would increase insight and stimulate interests of students.

History of mathematics helps a teacher in finding antecedents of one's work and gives deep understanding into the problems he/she himself/herself is studying or investigating. Thus it increases the width and depth of his/her understanding and assimilation in the area. It may suggest new connections between new and old ideas and methods.

New ideas often strike the mind while making historical studies and investigations. In this connection, it may be mentioned that Fermat got the idea of co-ordinate geometry when reading classical works of the past.

4.3 (b) Benefits to the students :

Among the many benefits to the students the following are noteworthy.

Study of history of mathematics helps in creating motivation, interest, excitement and stimulation among the learners. This will encourage reading and eliminate frustation, if any, for committing mistakes during study. A learner will be less discouraged by his/her mitake, as he/she learns that this does happen to great men.

The study of history of mathematics makes students aware of relations that exist between the society and ideas of culture and education. They get better understanding of theoretical mathematics and acquire a greater technical skill in its application.

Study of history of mathematics will eliminate negative attitudes which some students may bear to mathematics. One of the reasons may be ascribed to the esoteric, dry way in which the subject is taught. We tend to overwhelm our students with formulae, definitions, theorems and proofs, but we seldom mention the historical evolution of these facts, leaving the impression that these facts are handed over to us, like the Ten Commandments, by some divine authority. The history of mathematics is a goos way to correct these impressions'. By trying to interject some morsel of mathematical history or vignettes of people whose names arise in the context will lead to the growth of positive attitude towards trhe subject. When a subject is packed with stories of thrilling events and ancedotes, it becomes more interesting, more inspiring and more attrractive.

Study of history of mathematics would reveal to students with astonishment that there are lots of miscredits assigned. This will help them to try to correct the misplaced credits and reconstruct the history. Scientific achievements are a result of hard work, often austained and painful, of many scientists working over the centuries. It is our compelling duty to remember properly the actual pioneers and give due credits to their efforts. For example, the formula

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots\ldots$$

known as Gregory's series was known to India 250 years earlier than Europe. It would be proper to link the name of Madhava with this series.

Equation of the form $Dx^2 + 1 = y^2$, when D is positive integer but not a perfect square is known as Pell's Equation. But Pell has nothing to do with it or its solution. It's solution can be found by repeated application of principle of composition (Brahmagupta's Bhavana Principle). Bhaskar's 'chakravala' method makes the decisive step in determining the solution. So, it should be appropriate to designate the equation as the Bhrahmagupta - Bhaskara equation or even as the 'Bhasjkara equation.'

Ceva's theorem (17th century) was already known to A--Mm-Taman, King of Sargossa who ruled from 1081-1085.

Fibonacci sequence, Pythagora's theorem, Pascal's triangle, L'Hospital Rule, Argand Diagram, Horner's method etc. are all misnormers as they are not named after the first geniune discoverers.

Alos the study of history of mathematics will inspire students to develop international brotherhood and to realise that mathematics is not a national affairs but an international one and thus always has an international appeal.

More over, exposition of any subject or topic is not complete unless we know how it originated and how it has been developed. The history of mathematics helps us know these. Also, it is rightly said, 'History makes a man wise'. Conforming with the comment, history of science in general and history of mathematics in particular will make us even wiser. Besides, historical study is always rewarding.

Lastly, I agree with Hermann Weyl that 'perhaps mathematics is no more than history of mathematics.'

5. *Conclusion :*

From the above discussion we may easily conclude that the sooner we introduce history of mathematics in the mathematics syllabus the better, though there may be some debate about how much study materials on each topic may be included in the syllabus. An attempt has been made here to give a brief sketch of the syllabus of history of mathematics (mere Proposal) at the higher secondary stage and undergraduate stage which may be taken as a basis of discussion to give the final shape.

5.1 Proposed syllabus

(a) H.S. (10+2) Stage
  (i) Evolution of counting numbers and number system.
  (ii) Symbols and symboling.
  (iii) Evolution of Algebra.
  (iv) Evolution of Geometry.
  (v) Evolution of Trigonometry.
  (vi) Evolution of measure and mensuration.

(b) Undegraduate Stage
  (i) Evolution of theory of equations.
  (ii) Evolution of complex numbers.
  (iii) Evolution of two-dimensional and three-dimensional coordinate geometry.
  (iv) Origin and development of calculus.
  (v) Evolution of probability.
  (vi) Origin of LPP.
  (vii) Origin and development of computer science.

References :

1. Bhanu Murthy, T.S. (1992) :- A Modern Introduction to Ancient Indian Mathematics; Wiely Eastern Ltd.
2. Bhatnagar, S.C. (1989) : - the Status of Mathematics Teaching in Academia; Mathematical Education, Volume 5, No. 2 (pp 95-98)
3. Carss, M. (ed.) :- Proceedings of Fifth International Congress on Mathematical Education held in Adelaide in 1984; Birkhauser, Boston.
4. Cajori, F. (1919) :- A History of Mathematics (2nd Ed.); The Macmillan Company, New York.
5. Eves, H. (1956) :- An Introduction to History of Mathematics; Rinehart and

Co., New York.

6. Fashesh, M. :- Mathematics, Culture and Authority : A Unseco Project Report.
7. Gupta, R.C. :- Relevance and Importance of the History of Mathematics to Mathematics in Present Day India; The Mathematics Teacher, Vol.22 issues (3 & 4), (p. 14-18)
8. Kapur, J.N. (1969) :- Essays On Mathematical Education; Ram Chand and Co., New Delhi.
9. Maor, E. (1999) :- e : the story of a Number ; Universities Press (India) Ltd.
10. Ministry of H.R.D. (1986_ : National Policy of Education - 1986; Government of India.
11. NCERT (2000) :- National Curriculum Frame Work for School Education : A Discussion Document, NCERT, New Delhi.
12. Sat, S. (2002) :- Pedagogical aspects of representation of a mathematical concept : the teaching of complex number; Jour. of CPSM, Kolkata (12th issue) (p. 14-21)
13. Sar, S. (2003) :- Evolution of a few algebraic Symbols ; The Mathematics Teacher, AMTI, Chennai Vol. 39, issues (3 & 4) (p. 201-206).
14. Sar, S. (2003) :- Ganiter Shaka Prashakha (in Bengali); Jnan Bichitra Prakashani, Agartala.
15. Smith, D.E. (1958) :- History of Mathematics (Vol. I & II); Dover Publication, New York.
16. HPM (2001) :- The International Study Group on Relation between History and Pedagogy of Mathematics : The First Twenty Five Years, 1976-2000; HPM News Letter.
17. Zweng, M. and Others (editors) (1983) :- Procesing of Fourth International Congress on Mathematical Education held in Berkley in 1980; Birkhauser, Boston.
18. Bell, E.I. (1945) :- The Development of Mathematics ; McGraw Hill Book Co., New Yorlk.

The Mathematics of Some Physical Situations

★ **Pijus Kanti Das**

Statement:

Why do we flex our knees while jumping from a certain hight to encounter lesser resistance?

Proof:

Let the mass of the man = M

R_1 = Resistance offered by the ground if the man flexes his knees.

R_2 = Resistance offered by the ground on the man if he does not flex his knees.

T_1 = Time of coming to rest from the moment he touches the ground with flexing his knees.

T_2 = Time of coming to rest from the moment he touches the ground without flexing his knees.

U = Initial velocity of the man at the moment just before he touches the ground.

Case - 1 With flexing his knees.

Impulse = Change in momentum.

i. e., = R_1T_1 = M(Initial velocity - Final velocity)

= M(U-0) = MU

i. e., $R_1T_1 = MU$ .... (i)

Case - 2 Without flexing his knees.

Impulse = $R_2T_2 = M(U-0)$

i. e., $R_2T_2 = MU$ .... (ii)

Since, change in momentum remains same in both the cases

$\therefore R_1T_1 = R_2T_2$

or, $R_1 = R_2(T_2/T_1)$ .... (iii)

If the man flexes his knees, then the time taken to come to rest from the moment he touches the ground is more than the case withuot flexing his knees.

i. e., $T_1 > T_2$

From (iii) $R_1 < R_2$

Hence, resistance offered by the ground with flexing his knees is less than the resistance offered by the ground with flexing his knees.

★ *Asst. Teacher, Bangur High School, Tollygunge, Kolkata*

Illustration :-

Prob :

Let us assume the following data:

A person of 25 kg. jumps from a hight of 6.4 m and is brought to rest within 2.0 sec flexing his knees, he would have come to rest in 0.1 sec. Had he not flexed his knees, what forces will be extracted on his body in the two cases? :

Solution : Mass of the men = M = 25 kg

Velocity of the person just before touching the ground,

$$U = \sqrt{2 \times g \times 6.4} = 11.2 \text{ M/S}$$

Impulse in the case = Change in momentum

i. e., $R_1T_1 = MU/T_1 = (25 \text{ X } 11.2)/2 = 140$ N

Again impulse in the 2nd case

$R_2T_2 = M(U\text{-}0) = MU$

$\therefore R_2 = MU/T_2 = (25 \text{ X } 11.2)/0.1 = 2800$ N

Thus $R_2/R_1 = 2800/140 = 20$

or, $R_2 = 20\ R_1$

i.e., In the 2nd case (without flexing)

20 times more resistance is exerted by the ground

Probing question :-

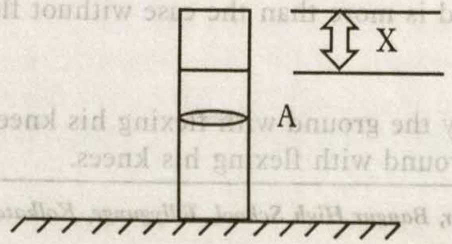
Why does a goalkeeper or a cricket fielder catch a ball & pull it through a distance instead of stopping it immediately?

Critical analysis :-

Topic :

Stress (or strain) produced by a suddenly applied load is double than that produced by a gradually applied load.

suppose, a load W is applied onto a bar of uniform cross section and is extended or compressed (as the case may be) by an amount X and the corresponding stress induced be f_1 for gradually applied load & f_2 for suddenly applied load respectively.



Case I

When load is applied gradually

Work done on the bar = Strain energy stored in it

$((0 + W)/2) \times X = \frac{1}{2}(f_1 A) X$

or, $f_1 = W/A$ .... (1)

Case II

When the load is applied suddenly

Work done on the bar = Strain energy stored in it

$W \times X = \frac{1}{2}(f_2 A) X$

or, $f_2 = 2W/A$ .... (2)

From (1) & (2) we see $f_2 = 2 f_1$. Hence proved.

Mathematical Explanation :

Let,

R_1 = Resistance offered by a hard floor on a man when he is let fall from a certain height.

R_2 = Resistance offered by a standy ground on a man when he is fallen from the same height.

T_1 = Interval, during which the man comes completely to rest falling on a hard floor from the moment he touches ground.

T_2 = Interval, during which the man comes completely to the rest falling on a standy ground from the moment he touches the ground.

U = Velocity of the man at the moment just before touching the ground (either hard floor or standy ground)

Case I :

Falling on a hard floor

Impulse (I) of R_1 is given by,

$R_1 T_1$ = Change in momentum of the man

= M((Initial velocity of the man) – (Final velocity of the man))

i.e., $R_1 T_1 = M(U-0) = MU$

i.e., $R_1 T_1 = MU$ .... (1)

Case II :

Falling on a standy floor

Impulse (I) of R_2 is given by,

$R_2 T_2$ = Change in momentum of the man

= M((Initial velocity of the man) – (Final velocity of the man))

$= M(U-0)$ .... 2)

Since, momentum change in both the cases are equal to MU.

$\therefore R_1T_1 = R_2T_2 = \text{Constant} = MU$

or, $R \propto 1/T$

Hence, resistance offered by any type of ground (whether a hard floor or a standy one) on a person when he is fallen from a certain height is inversely proportional to the time during which he completely comes to rest from the moment he touches the ground.

Molecular Theory :

from the concept of imperfection of crystals :

When an external force is applied on a body, it may be subjected to dislocation i.e., orientation of the crystal latice will be changed.

If the external force is applied rapidly then the dislocation valocity becomes more because of very small interval of time and therefore straining becomes large and hence more stress is developed.

If on the other hand the external force is applied gradually then the dislocation velocity becomes less because of considerable interval of time and therefore less stress is developed.

Probing Questions :

Why is a man hit harder when he falls on a paved floor then when he falls on the sand from the same hight?

Ans :

(A) Considering time of coming at rest

The man comes at rest within a very short interval of time owing to the non displacement of the hard(paved) floor and hence the floor offers more resistance to stop him within a very short interval of time.

When the man on the other hand, falls on the sand, the sand is displaced more & it takes more time to stop him and hence it offers lesser resistance on him.

(B) Considering displacement :

When a man falls on sand, the sand also offers resistive force and therefore, the man is stopped after being displaced to some extent into the sand

Let, the man falls from a height H

M = Mass of the man

X = The man is displaced along with sand against the resistance of sand from the moment he touches the ground.

R = Resistance offered by sand on man.

H = Height from which he falls

Using the concept work–energy relationship,

Work done against resistance = Loss of K. E of the man

$$(R - Mg)X = MgH$$

[∴ KE at the of the man moment just before he touches the ground = P.E at height H]

$$R = (1+(H/X))\,Mg$$

X being more in case of sand the resistance becomes less and therefore the man feeels less pain.

Geometrical Significance of New Derivative

* Asim Mukhopadhyaya.

1. *Introduction :*

'If change is the only truth of the universe, then derivative is the only science to unevil it', this is an excerpt from the opening paragraph in my article, DERIVATIVE, published twenty years ago in October, 1983 issue of Junior Science Digest, New Delhi. The article dealt with the derivative of a function at a point from first principle and observed that the process induces a qualitative change in the quantitative values with the function itself. Author advanced a short poem composed by him to describe the successive steps to obtain derivative of a function from first principle :

Arbitrary h is chosen,
So that $x + h$ is in the domain.
Take the difference of $f(x)$ from $f(x + h)$.
And divide the difference by h.
Stop not! But send h to zero.
Let not the ratio turn a hero,
But to a value positive, zero or negative,
What Isaac Newton described as derivative.

The most enchanting constroversial point as to the discovery of calculus whether by Sir Isaac Newton of England or by Gottfried Wilhelm Leibnitz of Germany had been nicely discussed with some remarkable notes offered by Frederich Engels in his famous book Dialectics of Nature. The contributions of predecessors of Newton and Leibnitz to the development of the subject were referred to with special importance by Sir Isaac Barrow, teacher of Newton at Trinity College and who vacated the post of his professorship in favour of Newton, a man then 28 years old, before completion of his tenure on the ground that a greater sun (Newton) had approved. Moreover about 500 years before Newton and Leibnitz, Bhaskaracharya II (114-1185) knew some basic principles of calculus, because in his various types of trigomometrical results he succeeded to derive a result like $d(\sin\theta) = \cos d\theta$, by which he was able to forecast the position of the sun in the sky at certain periods of a year.

Besides advancing the derivatives of parametric equations and that of a function with respect to another function both from first principle, the

---

* *Ex, Reader of Mathematics, Charuchandra College, Kolkata – 29*

prementioned article was enriched with a survey on various types of derivatives, namely, Dini's derivative, Schwarz derivative, Strong derivative due to Esser and Shisha, Peano's derivative, Roussel's derivative, Approximate derivative due to Denjoy, Fractional derivative and the like. In this context, names of some eminent Indian mathematicians, so far known to the author, P.C. Bhakta (Jadavpur University) and Dr. B.K. Lahiri (Kalyani University), who made masterly ontributions in the landscape, had been highlighted in the article. In conclusion, a new type of derivative, called 'new derivative', drawn by the author himself, was introduced, which is now dealt with in the following section.

2. *New Derivative*

Let $y=f(x)$ be a finite and single-valued fiunction defined in any interval of x and assume that x is any interior point in the interval, then the 'new derivative' of $f(x)$ at that particular value of x is defined as

$$\underset{h\to 1}{Lt}\frac{f(xh)-f(x)}{h-1}$$

provided the limit exists.

It is very interesting to note that by the above principle the derivative (new) of x^n, $\sin x, e^x$ etc. are respectively nx^n, $x\cos x$, xe^x etc. In generl, we have the following relation.

new derivative $= x$ ordinary derivative

$$= xf'(x)$$

Proof : New derivative $= \underset{h\to 1}{Lt}\dfrac{f(xh)-f(x)}{h-1}$

$$= \underset{k\to 0}{Lt}\frac{f(x+xk)-f(x)}{k}\text{, by putting } k=h-1$$

where $k\to 0$ as $h\to 1$

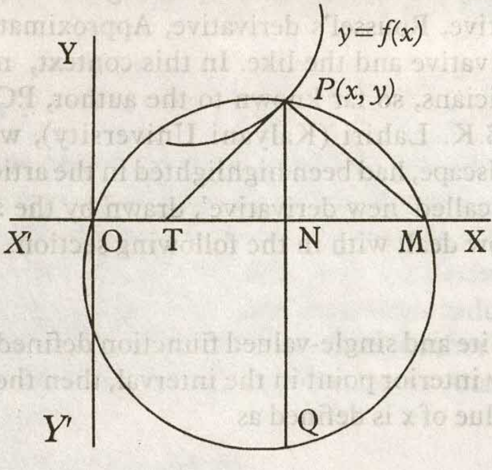
$$= \underset{xk\to 0}{Lt}\frac{f(x+xk)-f(x)}{xk}x$$

$$= f'(x).x = xf'(x)$$

Further, defining integration as anti-derivative or as inverse process of differntiation, a new type of integral calculus may be developed.

The idea may have crossed readers' mind as to the feasibility of such derivative. The great advantage of ordinary or Newtonian derivative is that it has a geometrical significance. The present article poses to advance some geometrical significance of 'new derivative'.

3. Geometrical Significance



Let the tangent and normal at a point $p(x, y)$ on a curve $y = f(x)$ meet the axis of X at T and M respectively, so that TN and NM are the subtangent and subnormal to the curve at P, where PN is the ordinate of the point P.

Now new derivative $= xf'(x)$

$$= \text{ON. } \tan\angle\text{PTN} = \text{ON.PN} / \text{TN}$$

or, TN. new derivative $= \text{ON.PN.}$

Thus new derivative at a point on a curve geometrically signifies a line segment such that the rectangle contained by it and the subtangent at the point is equal to the rectangle contained by the abscissa and the ordinate of the point.

Further, PN be produced to meet the circle through O, P, M, at Q, then ON.NM = PN.NQ

But new derivative $= xf'(x)$

$$= \text{ON.} \tan\angle\text{PTN}$$

$$= \text{ON} \tan\angle\text{NPM}$$

$$= \text{ON.NM/PN} = \text{NQ}$$

Thus, alternatively, new derivative at a point on a curve stands for the portion of the ordinate at the point by which it stands for the portion of the ordinate at the points by which it should be extended, so that the points O,P,M,Q are concyclic.

Some Studies Related to Children's Learning of Mathematics

* Sanghamitra Gayen

1. *Introduction*

Mathematics is one of the most important subjects in the school curriculam. Its study is a vehicle to train a child to think and reason precisely and efficiently. At primary stage children learn arithmetic besically about the four operations with whole numbers. However, it is noticed that students often find difficulty in understanding further arithmetic or display a dislike for the subject and develop a negetive attitude towards mathematics. One of the reasons for this repugnance could be that arithmetical skills acquired in primary school are just skills and may not involve understanding of the process. As a result, they may not be able to relating what they study to their environment or their daily life.

2. *Cognitive development and mathematics learning*

Cognitive development according to Piaget's theory involves the growth of knowledge, and knowledge includes not only information but also the rules for organizing and transforming information. Schema in a psychological term is used for denoting mental structure. Odaka Tashio in his schema formation theory in mathematics learning stated that learning mathematics means mathematical schema formation process embedded in a 'situation–behaviour'. This term includes not only complex conceptual structures of mathematics but also simple structures which co-ordinate sensory motor activity.

Piaget stated that conservation of number is generally achieved by the age of 6 to 7. But complete understanding of number requires some comprehension. Many children [1] are not able to understand subtraction as the reverse of addition because they do not yet have the idea of Piagetion concept of reversibility. Ramesh Chandra Kothari [2] in his work found that only 62% pupils could solve daily life problems based on subtraction. It was also observed that lack of proper methodical approach was one of the factors for low achievement in mathematics.

However it is necessary that children must have much experience of physically manipulating sets, and discovering properties of set operation.

3. *Language development relating to mathematics education*

At the lower primary stage the learner should master basic vocabulary and symbolism related to numbers, geometrical objects, money transactions etc. But

---

* *Lecturer, Haldia Govt. College, East Midnapore*

reality of situation in school may be quite different. Another reason for the lack of understanding of arithmetic may be lack of language skills. Children come from a varity of backgrounds, and formalized language of the teacher and books may pose an intimidating barrier towards assimilation of the ideas offered to them.

A study by N. Swarnalekha revealed [3] that improvement in the problem solving area of mathematics was attained by paying attention to the language comprehension skills. The work of G. C. Upadhaya et. al [4] revealed that vast majority of children lacked adequate levels of readiness for reading and number work at the time of entry to school.

4. *Task ahead*

Thus it is necessary to investigate whether language ability of children in primary school affects their understanding of basic arithmetic. The relevent variables in this connection are numarical skill, numarical understanding and language comprehension. It is extremely important and urgent to find the interrelationship and interdependance between them. It is expected that the investigation would shed extra light on the arena of learning primary mathematics.

5. *References*

[1] Cruikshank, E. D. et. al - Young children learning mathematics, Allyn and Bacon Inc. 1971

[2] Kothari, R. C. - Minimum level of learning based approach in teaching mathematics, Indian Education Abstructs. Issue – 6, Jan-1999, N. C. E. R. T.

[3] Swarnalekha, N. - Teacher empowerment strategies to promote problem solving ability in mathematics through joyful active learning, Indian Education Abstructs, Jan-1997, N. C. E. R. T.

[4] Upadhaya, G. C. et. al - Numeracy and reading readiness levels of entrants to class – I, Indian Educational Abstructs. Issue – 6, Jan-1999, N. C. E. R. T.

* The author is grateful to Dr. Adti Ghosh, Reader, C.U. for her kind advice and guidance in writing this paper.

A Review of Studies Regarding Acquisition of Geometry by School Students

*** Aditi Ghosh, ** Chandrani Acharya**

Introduction

Mathematics and in particular Geometry, is not always a popular subject at the high school. Nevertheless Mathematics and with it Geometry is persued by countless high school students for the purpose of acquiring their dreams about sucessful professions. However, their comprehension of the basic ideas regarding Geometry and accompanying visual-spatial skills is belied in their actions and approaches to problem solving.

People in general too, face situations and require to solve problems that involve visual-spatial skills and basic Geometric ideas. Some, like the potter, train themselves over the years, often unconsciously, in skills that involve basic Geometric ideas. Others are often seen completely inadept when it comes to using their visual-spatial skills.

While there are plenty of students who are very able regarding Geometric skills and concepts, it may be noted that plenty of students at class – X and later stages do not have concrete bases for the Geometry that they study. These include not only mathematical ideas but other tasks that have applications of these ideas as well. If this is indeed so, then there is a need to consciously look at the primary and upper primary curricula so as to ensure that these ideas are thoroughly imbibed by children before they enter high school. this paper, therefore, looks at the different studies in this field so that the modern aspects of geometry teaching and learning may be more pertinent and useful.

What reqires further inquiry is how far do different environments affect the acqui à-vis basic Geometric ideas. Children from urban and rural environments, come with different life experiences. So do children who come from educated homes and those who do not. On the other hand much can be said of the child who may come from the uneducated potter's family. The latter may come enriched with a different state of visual-spatial skills, which have possibly become an inherent trait in his/her personality. Therefore, the impact of different environments and their accompanying material and cultural baggage is a question that may occur to the educator.

** Dept. of Education, Calcutta University, ** B.Ed Dept. Scottish Church College, Calcutta*

Physical Space, Geometry and Improvement of Visual - Spatial Skills

Van Heilie, in his paper "The thought of the child and Geometry" [10] suggest five levels of thought development in Geometry and identifies some of the faults with current Geometry programmes.

Recent developments on Brain Theory [10] rely on two important findings :-

- The human brain has two hemispheres.
- Each hemisphere is specialized for different modes of thought.

The left hemisphere is to do with language, thinking in words etc. The right hemisphere is to do with spatial, musical, artistic, symbolic, creative thinking etc.

Most infants are right brain oriented. Current mathematics preogrammes do little to nurture and develop space concepts. Students can accept and use only those axioms that they understand and have experienced in their real world [10].

The work of M. C. Mitchelmore of University of West Indies [15], is perhaps very important in the study of development of the children's spatial ideas. According to him young children's conception of space is dominated by topological and local relationships that they are unable to coordinate.

Campbell [6] found that there is a strong relation between the performance on two dimension and three dimension drawing tasks. Both the skills develop as a result of maturation. The results also suggest that it is doubtful whether young children are infact attempting to represent perpendiculars in thair three dimension drawings and ignoring the parallels.

Perhaps the most interesting biological description has been given by Brian Butterworth regarding the relation of human organs to the spatial skills [5]. According to him, the primary visual cortex is connected to a special organ which has evolved for detecting visual features of the world. Language also plays an important role. Damage to the human parietal lobe can result not only in difficulties with numbers but also in problems with orientation in space, with control of actions and with representations of one's own body.

Gerstmann [8] himself thought that numbers and space are related functionally. As few as 14% of pupils think of numbers in spatial terms. For most people who do see a line, it goes from left to right though some report right to left and for others the line goes straight up.

Effect of Cognitive Development on Mathematical Achievement –

The cognitive development levels differ among boys and girls and also have a great impact on the aquisition of various skills, specially visual-spatial skills. Various studies have been conducted by a large number of researchers and significant correlations between cognitive development and achievement level of the child have been found [12].

Mahapatra, Madhuri [14], assessed the cognitive development level of the primary stage, pupils with special reference to gender and found that at the higher level of cognitive development, the percentage of boys was found to be higher than the girls. Pupils of the same chronological age did not belong to the same cognitive development level.

Thirumurthy, Vidya [20], studied the children's cognition of Geometry and spatial reasoning as a cultural process and explored the indigenous cultural practices that shape Geometry and spatial thinking in young children. Six families from two different cultures, three Japanese and three Maxicans, were selected from the North West corner of the United States of America. The focus was on the socio-cultural contexts that have the potential to guide the cognition of Geometry and spatial reasoning of children in the age group of four and seven. The finding demonstrates that the mothers with higher educational lavels created contextualized cultural experiences for their children. The parents mediated the learning through language and guided them using verbal instructions and modeling.

Some findings indicate that language reinforced with visual cues, increases effectiveness of teaching. This is specially pertinent in teaching basic ideas regarding Geometry.

According to Cognitive Load Theory (Sweller, Van Merrienboer and Paos, 1989) and Mayer's Theory Of Multi media learning, replacing visual text with spoken text (the modality effect) and adding visual cues relating to elements of a picture to the text (the cuing effect) both increase the effectiveness of multimedia instructions in terms of better learning results [19].

Huib K. Tabbers, Rob L. Marthens Joroen J. G. Van, Merrienboer, studied the generalisability of the modality and cuing effect in a classroom setting and found that adding visual cues to the pictures resulted in higher retention scores while replacing visual text with spoken text resulted in lower retention and transfer scores [19].

Spatial Abilities Realeted to Various Factors of The Environment –

Much of this research has been guided by Alan J. Bishop's [2] concern with teaching to understand moer about how one can help children to improve their spatial abilities in a mathematical context. As a result of more detailed testing and analisys, the feeling grew that, the problem with students whose visualization abilities has been previously assumed to be weak, was infact their lack of knowledge and general unfamiliarity with, the many spatial and visual conventions which we use in spatial and geometrical test materials.

According to him, there are two kinds of abilities :

1. The ability for interpreting figural information (IFI).
2. The ability for visual processing (VP).

IFI involves knowledge of the visual conventions and spatial vocabulary used in geometric work, graphs, charts and diagrams of all types. VP involves the idea of visualization, the translation of abstract relationships and nonfigural data into visual terms, the manipulation and exploration of visual imagery and transformation of one visual image into another.

Finally it may be provoking to consider whether the typical mathematics classroom is the best place to develop and teach VP ability with its heavy reliance on text book, board work and exercise. He feels that the reason many children turn away from mathematics is their dissatisfaction and unhappiness, with an excess of logico-analytic method. He thinks that many of them are predisposed more towards VP but teachers and texts are not stimulating or rewarding that ability [2].

John J. Del Grandy, Canada [9], studied on spatial and visual aspects and considered that spatial abilities such as "Perceptual Constancy", "Perceptual Abilities", "Perception of Spatial Relationships", "Visual Discrimination", could be enhanced and improve through an appropriate geometric programme. Geometry should be an integral part of elementary mathematics at the K. G. level, relying on the child's intuitions about space and experiences of movement of objects in space.

Peter Bender [3] indicated a psychological basis for continuity being a natural essence of mental-spatial operations, and also a natural part of the notions of rigid body coming under the heading of Congruence Geometry.

Piaget's concept of quantitative thinking in children was systematically replicated for Indian children from the city of Hyderabad. The results of the study pointed out that age, the type of quantity and the type of material significantly influenced the development of quantitative thinking. Sex and socio-economic background did not effect success in a quantitative thinking. There was a low positive correlation between IQ points and the scores on quantity comparison [1].

Singh J. and Srivastava S. N. [17], investigated the impact of parent's literacy on the academic achievement of 85 first grade children (aged 6 - 8 years) and 85 fifth grade children (aged 10 - 13 years) who were divided into two groups: those with illiterate parents. The subjects' scores on an achievement test indicated that while the younger subjects' academic behaviour may have been influenced by parental literacy or illiteracy, the older subjects' scores were affected by several other factors.

The impact of familial deprivation [18] on the aquisition of skills for the pictorial depth perception was studied on two groups of 125 children each from

Indian nurseries and orphanages, ranging in age from $3 - 6\frac{1}{2}$ years. analysis of the mean sports revealed a clear development trend in both the groups. Lack of heterogenetic and absence of stimulation in orphanages had a general retarding influence on the development of skill for pictorial depth perception.

Summarizing the results of different studies made by Reisen (1961), Bruner (1959), Brownfeild (1964), Zubek (1969) on the effect of deprivation and isolation conditions, Bruner [18] has emphasized the role of improverished environment, lack of heterogeneity, and reduced opportunity for manipulation and discrimination in the impairment of cognitive growth. Research trends point to the importance of the child's environment which provides the experimental basis for the acquisition and proper development of various cognitive skills.

Studying the impact of intelligence, education and isolation from the dominant cultural norm, Hudson [11], concluded that in the acquisition of skill for depth perception "the critical threshold is cultural and not educational."

Bobora [4], found that the academic achievement of the children in threw income levels were significantly different. A high significant difference was also found in attitudes of the parents of 'First Generation Learners' and 'Not First Generation Learners.'

Minimum Level of Learning at Primary Stage –

The report of the committee set up by the Ministry Of Human Resources Development, Government Of India, regarding "The Understanding Of Geometrical Shapes And Spatial Relationships", suggests that at the end of class-V the children should be able to draw triangles, rectangles, squares with the help of rular etc. They should also draw a circle with a given radius, know various terms related to a circle and their relationships, identify whether a pair of simple figures are reflections of each other or not and can draw the line of reflection if it exists [16].

Lowenfield [18] analyzing children's drawings, demonstrated that spatial relations are grasped step by step. No organized spatial relations were found in the youngest age group of 4 to 7 years. Between 7 to 9, some awarness of spatial order is evident; it was only by 11 to 13 years that Lowenfield observed full awarness of perspective.

Piaget [13] has shown how, in his view, the child slowly builds up the ideas of the 'object' during the first two years of life. During this this period he gradually comes to distinguish between his own body and other objects of the environment, and builds up a picture of the world as consisting of a number of objects that continue to exist even when they disappear from his sight and he learns that, as a rule, they maintain their shape and size.

According to Piaget [7] the ability of classifications develop by grouping things together and by taking into consideration their similarities and equivalence. The children learn to comprehend the relationship of the whole to its parts. Such behaviour is clearly classificatory since there is a class inclusion. The concepts of simple and multiple classification develop when the child is able to group objects according to a single dimension and multiple criteria respectively.

Bibliography –

1. Bhogle, S. and Fatima, T. Development of quantitative thinking in children.
   Social Change, 11 (2), 1981, 13 – 20
2. Bishop, A. J. - Spatial Abilities and Mathematical Thinking
   In Zweng M. et al (Eds), Proceedings of The Fourth International congress On Mathematical Education (ICME), Berkley, California, Birkhauser, 1983 (176 – 178)
3. Bloom, W. R. et al
   Spaces and Geometry
   In Hiest, K. et al (Eds)
   Proceedings of the Sixth ICME, ICMI SECRETARIAT
   Jawos Bolany Mathematical Society, 1991
4. Borbora, R. D.
   The impact of socio-economic status on the academic achievements of the First Generation Learners
   Indian Educational Abstracts, Issue-6, January 1999
   N. C. E. R. T.
5. Butterworth, B.
   The Mathematical Brain
   Papermac, Basingstoke and Oxford, 2000, (217 – 250)
6. Campbell D. T.
   Parallelogram Reproduction and the "Carpentered World" hypothesis: suggestion for cross-cultural research.
   Unpublished paper, North Western University, 1969 In Zweng M. et al (Eds)
   Proceedings of the Fourth ICME, Berkley, Calofornia, Birkhauser, 1983 (171 – 172)

Library

7. Gakhhar, S. and Kaur, R.

Development of ability to do Piagetian Conservation and Seriation Tasks at Concrete Operational State.

In Bevli, U. K. (Eds)

8. Gertsmann, J.

Syndrome of Finger Agnosia: Disorientation for right and left, Agraphia and Acalculia.

archives of Neurology and Psychiatry, 44, 1940 (398 – 408)

9. Grande, J. D.

Space as a Moddel For Elementary School Geometry.

In Zweng M. et al (Eds)

Proceedings of the Fourth International Congress On Mathematical

Education

Berkley, Calofornia, Birkhauser, 1983 (163 – 165)

10. Heile, V. P. M.

The Thought Of The Child And Geometry

In Zweng M. et al (Eds)

Proceedings of the Fourth International Congress On Mathematical Education

Berkley, Calofornia, Birkhauser, 1983 (163 – 165)

11. Hudson, W.

Pictorial Perception And Educational Adaption In Africa

Psychologia, Africana, 9, 1962 (226 – 239)

12. Kimura, D.

Sex And Cognition

A Brad Ford Book, The MIT Press

Cambridge, Massachusettes, London, England, 1999 (43 – 64)

13. Lovell, K.

The Growth Of Basic Mathematical And Scientific Concepts In Children

University Of London Press, 1971

14. Mahapatra, M.

Assessment Of The Cognitive Development Level Of The Primary Stage.

In, 'Studies On Classroom Processes And School Effectiveness At Primary Stage'

N. C. E. R. T. 1996

15. Mitchelmore, M. C.

The Development Of Children's Spatial Ideas

In Zweng M. et al (Eds)

Proceedings of the Fourth International Congress On Mathematical Education

Berkley, Calofornia, Birkhauser, 1983 (171 – 172)

EXECUTIVE BODY OF CPSM - 2005

PRESIDENT : DR. S. N. GIRI.
VICE - PRESIDENT : DR. G. K. DATTA.
SRI CHANDIDAS PAL.
SRI P. SALUI.
GENERAL SECRETARY : DR. M. M. CHEL.
JOINT SECRETARY : DRI H. S. MALLICK.
TREASURER : SRI A. K. BERA.
MEMBERS : DR. A. K. MONDAL.
PROF. K. C. MONDAL.
SRI NARAYAN BHATTACHARYA.
SRI ANIL KR. GAYEN.
SRI V. K. SRIVASTAVA.

Activities of CPSM at a glance

- Annual general meeting and delivery of lectures by distinguished teachers & teacher educators.
- State level Quiz, Quest & Talk contest in Mathematics.
- Seminar and workshop for mathematics teachers.
- Achievement - cum - Diagnostic Test in Mathematics.
- Paper reading and publication of research papers in state/national/international level.
- Action Research in Mathematics.
- Publication of Bengali Bulletin and English Journal.
- Different student programmes in regional/state level and prize distribution.

BOOK - POST

If undelivered please return to :
CENTRE FOR PEDAGOGICAL STUDIES IN MATHEMATICS
‘Regd. No. S/65536 of 1990-91
16, South Avenue, Kolkata - 75